The Performance of ADCP-Derived Directional Wave Spectra and Comparison with Other Independent Measurements

B. Strong and B. Brumley
RD Instruments
9855 Business Park Ave.
San Diego, CA, USA 92131
bstrong@rdinstruments.com

Dr. E.A. Terray
Department of Applied Ocean Physics and Engineering
Woods Hole Oceanographic Institution
Woods Hole, MA, USA

Professor Gregory W. Stone
Coastal Studies Institute and
Department of Oceanography and Coastal Sciences
Louisiana State University
Baton Rouge, LA, USA

I. Introduction

The measurement of waves, and in particular their direction, has been one of the more difficult problems in observational coastal engineering and oceanography. The need also to measure currents frequently confronts the practitioner with the necessity of deploying two instrument systems, such as a buoy and an ADCP. Because, in principle, ADCPs combine the required functionality to measure both waves and currents in a single compact package, there has been considerable interest in exploring their efficacy as a wave sensor. The pioneering work of Pinkel and Smith [1] and Krogstad et al. [2] demonstrated that a Doppler sonar using horizontally-projected beams could provide a high quality measurement of wave direction (see also [3]). However, because this approach does not yield the depth distribution of currents, we have pursued the use of upward-looking ADCPs employing a conventional “Janus” 4-beam configuration to measure both waves and currents. (Terray et al., [4,5,7], and Gordon et al., [6]). While these earlier contributions reported on various aspects of the problem, they were not comprehensive, and contained little comparison data to assess the performance of the ADCP against commonly used wave direction sensors, such as heave-pitch-roll buoys and pressure-velocity (PUV) triplets.

Over the past two years we have undertaken, in collaboration with a number of investigators worldwide, an aggressive program to validate the performance of conventional upward-looking ADCPs for measuring waves by means of field comparisons with traditional wave sensors. This article is a progress report on these efforts.

II. Theory of Operation

The operating principles by which wave information is extracted from upward-looking ADCPs are discussed in [4-7], but will be summarized here for completeness. The basic idea is that the collection of range cells along the various acoustic beams constitutes a sparse array of independent “sensors”, each of which measures the local instantaneous velocity projected along the beam. The auto- and cross-spectra of these signals in each frequency band are assumed to be known linear functionals of the directional distribution of the waves. Wave direction can be estimated by inverting this “forward” relation using the Iterative Maximum Likelihood Method [2, 4]. The elements of the cross-spectral matrix at any frequency (or wavenumber) contain directional information in both their phase and amplitude, and in this sense the ADCP lies partway between a pure array measurement, relying solely on phase, and a “point sensor”, such as a PUV gage.

Because the amplitudes of the along-beam velocities measured by the ADCP depend on direction, it can determine wave direction even when phase differences are too small to be of use (i.e. in very shallow water, or for waves whose lengths are well in excess of the maximum bin-to-bin separation). Apart from these special cases, however, over most of the wave band and water depths of interest, the many spatial lags provided by the array of ADCP bins permit the separation of multiple wave systems that differ in direction but overlap in frequency. They also tend to result in somewhat sharper directional distributions than are obtained from point sensors, such as PUV triplets and wave buoys, when these are analyzed equivalently.
The spectrum of wave height can be estimated from both the ADCP velocity measurements, and the direct echo-location of the surface along the four slant beams (the latter provides an estimate of the water depth as well). We have also found it useful to incorporate a pressure sensor into the ADCP, which provides redundant measurements of water depth and wave height

III. Performance Considerations

The goal of this paper is to demonstrate the accuracy of wave measurements obtained from ADCPs, and to validate a performance model.

A. Wave Height

As mentioned above, an ADCP equipped with a pressure sensor yields three independent estimates of the spectrum of surface elevation based on: pressure, velocity, and direct echo-location of the surface ("surface track"). This capability provides a strong internal consistency check on the non-directional spectrum.
In order to achieve agreement among these three estimates in an automated context for data collected under a wide range of wave climates, a number of error sources must be considered. These include uncertainties in both water depth and the height of the instrument above the bottom, the presence of currents, as well as the signal-to-noise ratios (SNRs) inherent in the three measurements (note that the equivalent height SNRs depend on frequency). A detailed treatment of these error sources and their effect on the different spectral estimates is beyond the scope of this paper, and will be discussed in a future publication. However, by taking these errors into account, we have been able to achieve good consistency between the various estimates. Examples of spectral agreement are shown below. Figure 1 shows amplitude spectra (in units of m/√Hz) computed from a single 20-minute burst collected with a 600 kHz ADCP in 17 meters of water. The average absolute difference between the minimum and maximum of the three spectra is 0.03m/√Hz.

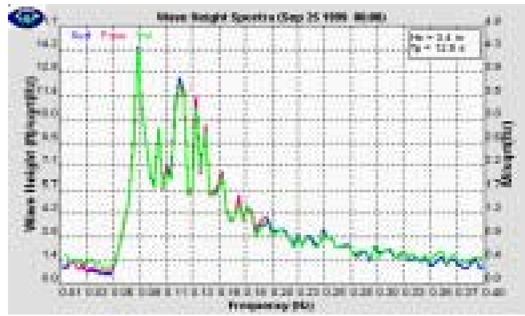

Figure 1. Wave amplitude spectra (m/√Hz) derived from pressure, ADCP velocity, and surface track measurements. Data is from Grays Harbor, WA.

To understand what part of this error constitutes a bias, rather than being due to sampling variability, we show in Figure 2 amplitude spectra derived from pressure, velocity and surface track, averaged over a period of 2 days. The average absolute difference between the three spectra is 0.02m/√Hz.

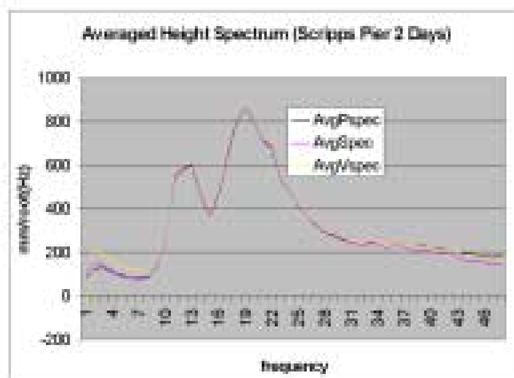

Figure 2. 1200 kHz ADCP deployed off Scripps Pier in 12 meters depth. Upper frequency is band 0.35 Hz.

Figure 3, below, shows the significant wave height Hs (defined here as 4 times the standard deviation of wave height) calculated by integrating the wave height power spectra determined from either pressure, velocity or surface track. The largest error occurs between surface track and pressure-derived estimates, and has a mean and standard deviation of 1cm and 2cm, respectively.

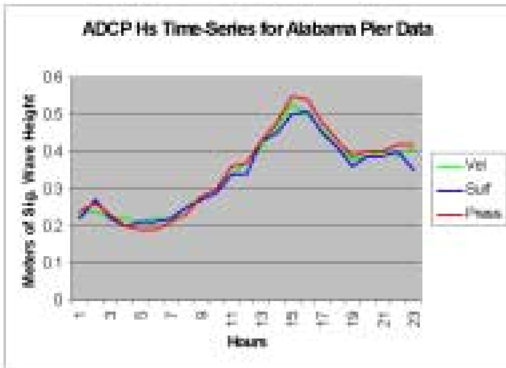

Figure 3. 1200 kHz ADCP data collected from a pier in Gulf Shores in 3 meters water depth.

Figure 4 shows the significant wave height Hs, defined as above, over a two month period, estimated from both the velocity-derived wave height spectrum, and directly from the time series of surface track heights. The mean and standard deviation of the error in this case are each 18cm (the average Hs over the entire period is 1.8m). Although the general agreement is good, the fractional errors are somewhat larger than in Fig. 3, because the surface track estimate (computed directly from the time series) uses the full signal bandwidth, and hence includes both additional noise and high frequency wave energy.

Figure 4. Significant wave height at Grays Harbor, WA, derived from the velocity spectrum and the surface track time series.

B. Wave-ADCP Performance Model

Each source of data (pressure, orbital velocity, and surface track) has its own short-term error due to measurement noise. The surface track also has quantization error introduced by the depth cell resolution. By taking the expected noise level for each measurement source and using linear theory to convert it to an equivalent *rms* wave height, we can estimate the minimum observable wave amplitude spectrum for the various methods. The result is shown in Figure 5 for amplitude spectra derived from pressure (light blue), velocity (light green), and surface track (yellow). The limiting frequency in each case is determined by the intersection of the underlying spectrum with the noise floor of each "sensor". To illustrate this, we have overlaid (dark blue) a schematic representation of a wave height spectrum.

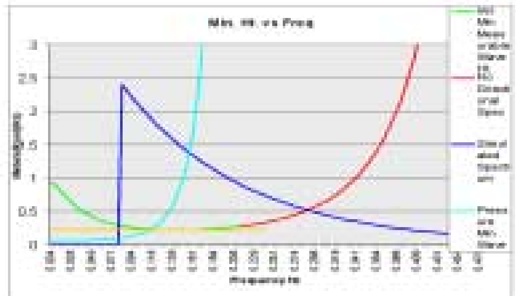

Figure 5. Wave model shows a set of expected performance curves for each technique.

As an example of how well this simple model works, we show in Figure 6 pressure and velocity spectra from a 600kHz ADCP in 42 meters depth compared to the predicted performance. Measured (brown) and modeled (light blue) pressure have the same cutoff frequency and slope. Likewise, measured velocity (dark green) matches the model (light green) well.

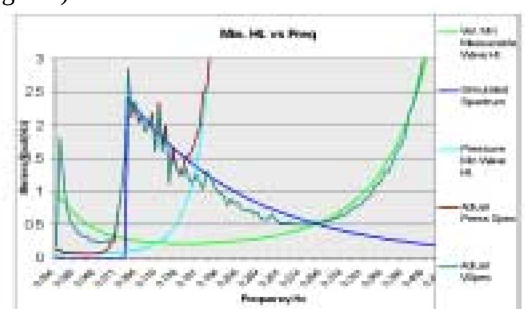

Figure 6. Actual data collected in 42 meters of depth is compared to modeled performance.

IV. Comparison Data

We now have data collected from Scripps Pier, CA., Grays Harbor WA, Duck NC, Denmark, United Kingdom, the east cost of Japan, Kyoto Japan, Cape Cod MA, and Gulf Shores AL. Most of these data sets have comparison data from independent instruments. In this section we examine a selection of these spanning a wide range of conditions.

A. Grays Harbor Data

This data set contains a fairly wide range of wave conditions, but can be characterized as mainly a Pacific coast wave climate – *i.e.* large, long period waves, SW to NW in direction.

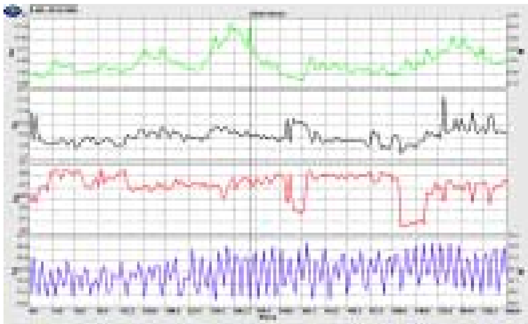

Figure 7. Time-series of significant wave height, peak period and direction and water level for Grays Harbor.

Comparison data were obtained from a co-located ADV at 17m depth, and a WaveRider buoy located in the vicinity of Grays Harbor. The pressure sensor used in the ADV was made by Paroscientific. Overall the ADCP-derived mean wave parameters compared very well with those estimated from the co-located ADV. The small differences observed are likely due to differences in their respective sampling schemes - the ADCP was sampled on a 4 hour schedule, whereas data were collected every 2 hours by the ADV. The data from the buoy, which was located somewhat farther away, are also in reasonable agreement with the ADCP.

1. Significant Wave Height

It is apparent from Figures 8 and 9 that the significant wave heights, Hs, computed from the ADCP are systematically higher than those from the ADV, and are more so at higher Hs.

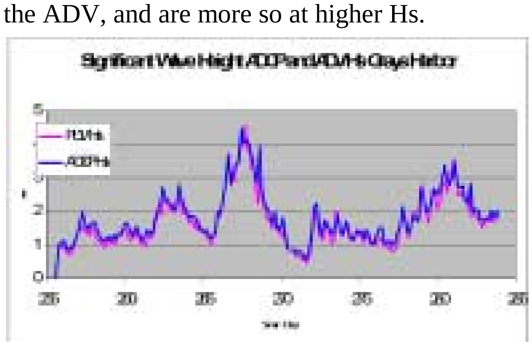

Figure 8. Time series of Hs for ADCP and PUV reference.

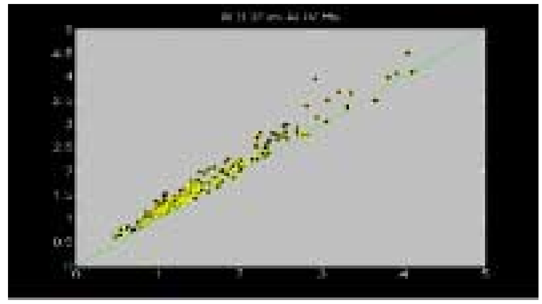

Figure 9. Scatter plot of Hs shows offset.

There are several possible reasons for the discrepancy. One is that the ADCP measures closer to the surface than the ADV, which was bottom-mounted. As a result, the cutoff frequency of the ADCP observations is 0.4 Hz, whereas it was 0.22 Hz in the case of the ADV. Hence the significant height derived from the ADCP data contains additional variance due to shorter waves. Figure 10 shows the significant heights calculated from ADCP and ADV spectra using the same cutoff of 0.22Hz. Although the agreement is better overall, there are still many points that differ significantly. Examining the spectra associated with these cases reveals that they occur at times when the tidal current is large.

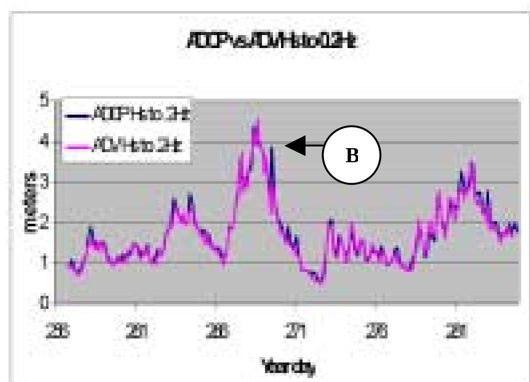

Figure 10. Differences between ADCP Hs and reference data remain even with identical cut-off frequency.

The worst case (denoted by B in the figure) corresponds to a current moving west against the incoming waves at 1 m/s. Such waves will be foreshortened, relative to their length in still water. Taking this effect into account, we can correct the ADCP wave height spectra derived from both pressure and velocity. These are compared in Figure 12 to the height spectrum from the surface-track, and are seen to show good agreement after correction.

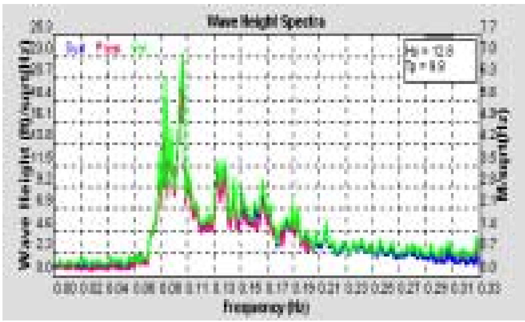

Figure 11. Good agreement among the height spectra derived from the ADCP velocity, pressure and surface track is obtained after correcting for the current.

The corresponding spectra without a correction for current are shown in Figure 12. Since the amplitude spectrum of wave height derived from pressure goes as σ^2/gk, where σ denotes the intrinsic wave frequency, and k is the local wavenumber (which depends on the current), we expect it to show a greater sensitivity to currents than the velocity-derived spectrum.

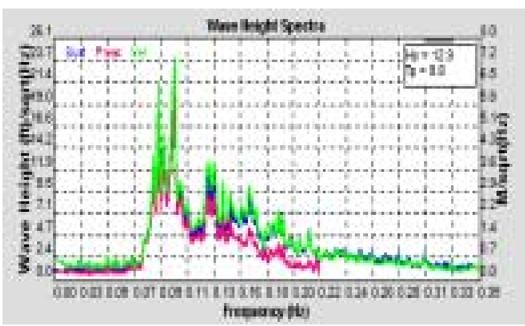

Figure 12. This is the same data as Figure 11, but pressure spectrum is in error without taking currents into account.

Table 1. Significant Wave Height Calculations for 09/26/99 at 2 PM.

ADCP Corrected for Currents with 0.35 Hz. Cut-off	3.95 m
ADCP Corrected for Currents with 0.22 Hz. Cut-off	3.87 m
ADCP Not Corrected for Currents with 0.22 Hz. cut-off, Using Pressure	2.87 m
ADV Not Corrected for Currents with 0.22 Hz. cut-off, Using Pressure	2.94 m
Buoy (not co-located)	2.46 m

Table 1 and Figure 13 show that the ADCP and ADV pressure measurements agree when no correction is made for currents.

We conclude from this exercise that pressure-based wave height spectra can be in error if a current-corrected dispersion relation is not used. A more detailed discussion appears in [8].

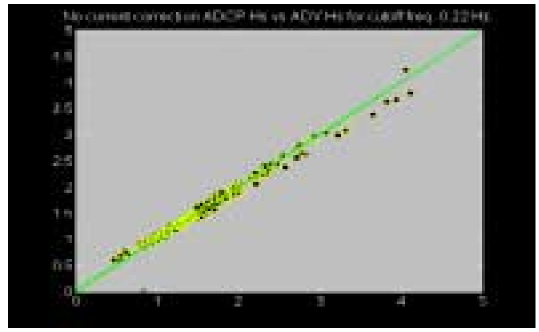

Figure 13. By neglecting currents, the ADCP and reference Hs measurements match.

2. High Frequency Wave Energy

Both wave pressure and velocity decay with depth (exponentially for sufficiently high wave frequencies). Hence the presence of noise necessitates that we impose a high frequency cutoff when extrapolating these quantities to the surface to infer wave height. The precise value of the cutoff depends on the noise level and the depth at which the measurement was obtained. Since the pressure measurement is taken near the bottom, whereas ADCP velocity measurements are obtained closer to the surface, we expect that the latter will have a larger measurement bandwidth. This is illustrated by Figure 14 in which we compare wave height amplitude spectra, derived from both ADCP velocity and surface track measurements, with that obtained from a bottom-mounted pressure sensor.

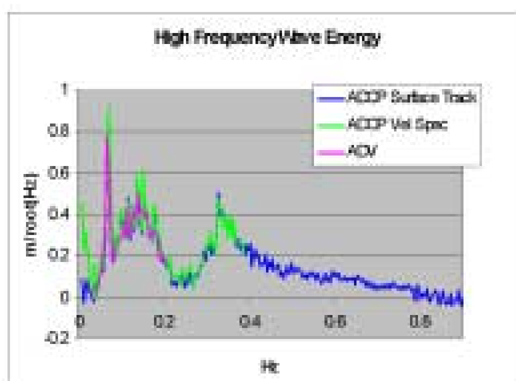

Figure 14. Local wind generates a spectrum at higher frequency (>0.25 Hz) than can be measured with a pressure sensor located on the bottom at 17m depth.

Note that at this depth (17m) the decay of the pressure above 0.15Hz is exponential with an exponent of $\sigma^2 z/g$. Hence the frequency cutoff for pressure is determined predominately by depth, and not by the accuracy or precision of the sensor. In 17 meters of water the frequency cutoff for pressure is only 23% higher if the pressure sensor is 2 orders of magnitude quieter. The benefit of surface tracking, and/or using orbital velocities near the surface is evident. The effect of the cutoff frequency as a source of bias in the estimation of Hs for the data shown in Figure 14 is summarized in Table 2 below.

Table 2. Error in Hs for Figure 14 due to the cut-off frequency.

	Surface Track	Orbital Vel	Pressure
Significant Wave Height	0.864	0.779	0.555
Cut-Off Frequency	0.9Hz	0.4Hz	0.2Hz
% Error relative to Surface	0.0	-9.7	-35.6

3. North and South Swell

Although this data set did not contain a good example of wave systems overlapping strongly in direction, the directional spectrum shown in Figure 15 shows some overlap, as well as illustrating the degree of directional resolution possible with the ADCP.

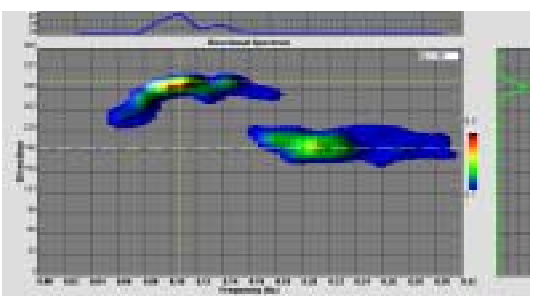

Figure 15. Swell from 2 directions.

The error in directional measurement associated with a PUV triplet is shown in Figure 16. The direction in the region where the North and South swells overlap (0.16 Hz) is smeared together in the PUV measurement, whereas the ADCP shows that there are no waves from 250 degrees but two wave systems from 201 and 310 degrees that overlap in frequency.

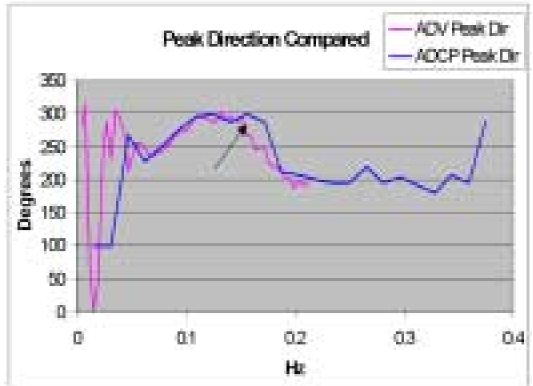

Figure 16. Peak directions for PUV and ADCP wave measurements at Grays Harbor.

4. Long Period Swell

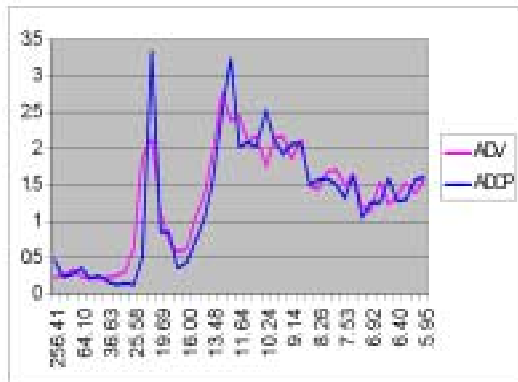

Figure 17. Height spectra compared for long waves (measured at Grays Harbor, WA, in 17m water depth)

That the ADCP can detect long period waves is evident from Figures 17 and 18, which show an example of long period (23s), narrow band swell seen by both the ADCP and co-located ADV at Grays Harbor on the Northwest Pacific coast. Although this wave system is fairly weak (it has an rms height of roughly 15cm), it is separated from the dominant waves in both frequency and direction. Its direction (from 233°) suggests that it may be the remnant of a southern ocean storm.

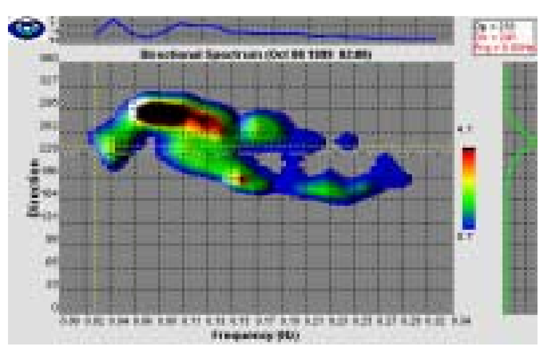

Figure 18. ADCP frequency-direction spectrum (of the data shown in Figure 17).

A 23s wave in 17m of water has a wavelength of 290m, so that the maximum bin separation (i.e. $2H\sin\vartheta_{Beam}$) is only 4% of the wavelength. Hence the maximum phase difference is less than 14° and it is at first surprising that the ADCP can determine the direction of such long waves. However it is straightforward to show [9] that, through 1^{st} order in the phase difference, the auto- and cross-spectra of velocities measured closest to the surface determine the directional averages of $\cos(n\vartheta)$ through n=4 and $\sin(n\vartheta)$ for n<4. Recalling that a PUV gage yields only the first two circular moments, this suggests that even for such long waves the ADCP can produce a sharper directional resolution.

B. WHOI 600kHz Data

These data were collected using a 600kHz ADCP bottom-mounted in 42 meters of water. They are typical of the wave climate off the eastern coast of the United States, which is predominately wind-driven, and either fetch or duration limited. The data set consists of 20-minute bursts collected every 8 hours from December 1998 through January 1999. It presents a nice contrast to the data shown earlier, both because the dominant waves are higher in frequency, and because it was collected in substantially deeper water. Comparison data were obtained from a co-located Seatex Wavescan heave-pitch-roll buoy.

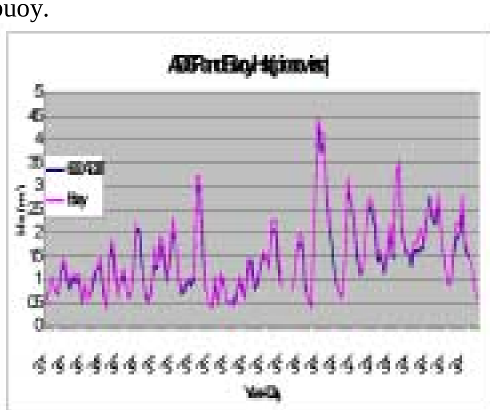

Figure 19. Comparison of significant wave height, Hs, derived from an ADCP at a depth of 42m, and an approximately co-located heave-pitch-roll buoy.

The data in Figure 19 are shown again as a scatterplot in Figure 20. This figure indicates the existence of a small bias. We note that the significant heights derived from the ADCP velocities and ADCP surface track matched well. Furthermore, measurement noise would tend to bias the ADCP estimates toward higher values. Although the source of the discrepancy is not know at this time, both of these considerations suggest that the ADCP estimates are reliable.

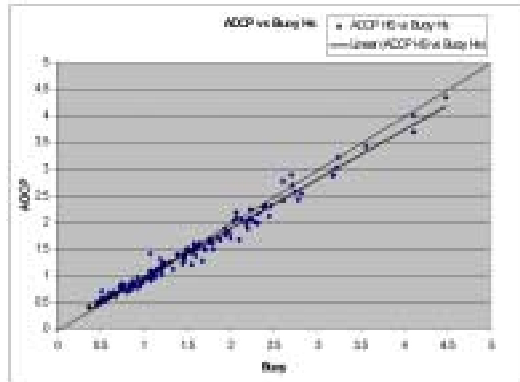

Figure 20. Scatterplot of ADCP and Buoy Hs.

Figure 21 shows time series and a scatterplot comparison between the ADCP and directional buoy estimates of peak direction. The agreement is quite good, provided that the same wave systems are selected in each case.

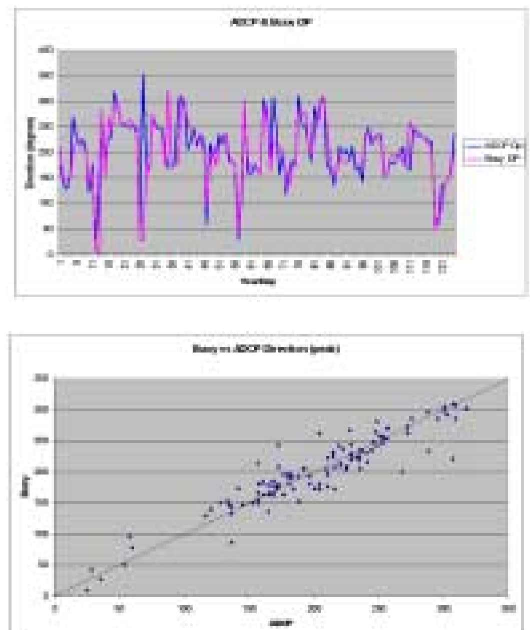

Figure 21. Peak direction measured by the ADCP and directional wave buoy.

The peak period, shown in Figure 22, is also in good general agreement between the ADCP and buoy. Discrepancies are almost always due to the selection of different peaks when there are several spectral maxima that are closely matched in height.

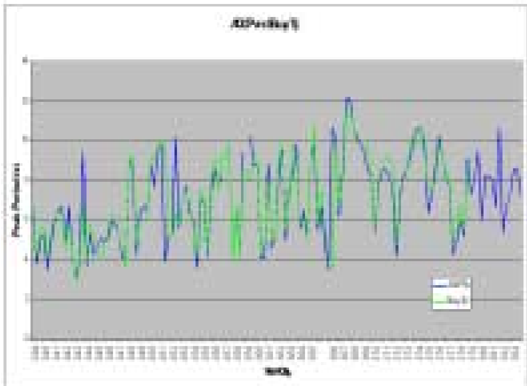

Figure 22. Peak Period for ADCP and buoy.

Figure 23 demonstrates the limitation of bottom-mounted pressure sensors for measuring short waves in deep water. The cutoff frequency for the height spectrum derived from pressure is 0.12Hz, and hence the pressure sensor entirely misses the more energetic wave system centered at 0.25Hz.

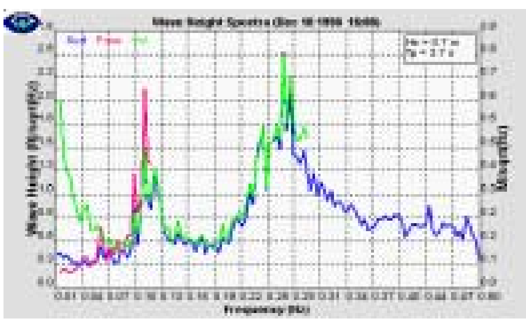

Figure 23. Amplitude spectrum of wave height derived from pressure (pink), ADCP velocities (green), and ADCP surface track (blue) for bottom-mounted instruments at 42m depth.

C. Danish Hydraulic Institute Data

This data set was collected with a 1200 kHz ADCP deployed off the western coast of Denmark in 7m of water. Comparison data were obtained from a bottom-mounted S4 directional (PUV) wave gage. However, this instrument was knocked over shortly after deployment, and there are only two days of coincident data available. The wave climate during this period consisted of multiple systems overlapping in frequency that arrived simultaneously from different directions.

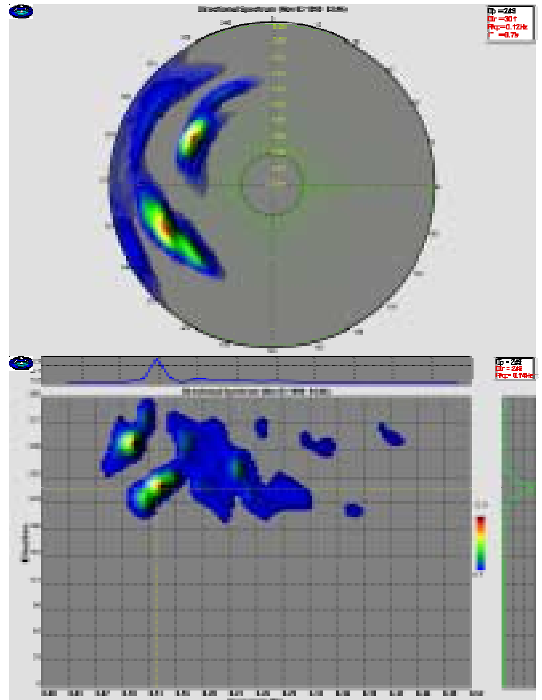

Figure 24. ADCP measured waves plotted in Cartesian and polar form show waves coming from different directions on Nov. 3, 4:00.

The ADCP data quality from this deployment was very high, and the various ADCP-derived estimates of the wave height spectrum were in excellent agreement. A typical frequency-direction spectrum from the ADCP is shown in Figure 24. The directional distribution for the ADCP data in Figure 24 is compared to that for the S4 in Figure 25. Another example from this data set is shown in Figure 26.

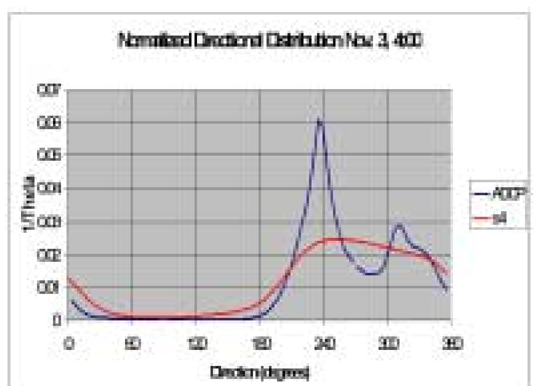

Figure 25. ADCP and S4 directional energy distributions compared.

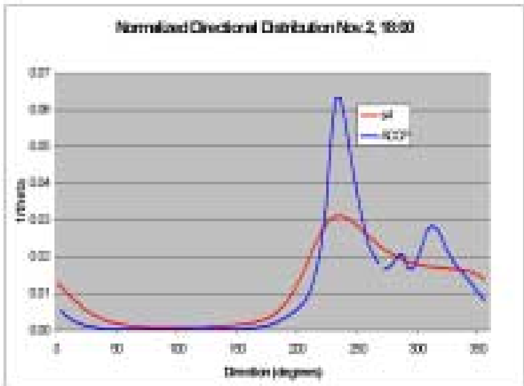

Figure 26. Two days of data indicate multidirectional energy distributions are badly spread for the PUV.

S4 directional spectra were computed using the Maximum Entropy Method. As expected, because it is a PUV measurement, and therefore yields only the first two (complex) circular moments of the direction spectrum, the S4 has difficulty in resolving different wave systems that overlap in frequency.

V. Conclusions

We have shown that a bottom-mounted, upward-looking ADCP provides a robust means of determining wave height and direction in coastal-depth waters. When equipped with a pressure sensor, the ADCP yields three independent estimates of the non-directional wave height spectrum, and hence provides an internal consistency check on the performance of the instrument. Directional spectra obtained from the ADCP tend to be sharper than those from point measurements, such as PUV triplets or directional wave buoys and, because of the greater number of degrees of freedom in the measurement, the ADCP can resolve complex multi-directional wave distributions.

Acknowledgments

The first three authors acknowledge the support of the National Science Foundation through an SBIR grant to RD Instruments that made possible our early work on this problem. ET was partially supported during the writing of this paper by NSF grant OCE-9811316, and ONR grant N00014-97-10483. We thank Drs. Mark Grosenbaugh and Jason Gobat at WHOI for their assistance in acquiring the data shown here as “WHOI 600”. Special thanks to Dr. N. Kraus of the U.S. Army Corps of Engineers for his support and access to the Grays Harbor data, and to Dr. K.Rorbaek of the Danish Hydraulic Institute for the use of his data.

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